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Article (Accepted Version)

Drenovak, Mikica, Ranković, Vladimir, Ivanović, Miloš, Branko, Urošević and Jelic, Ranko (2017) Market risk management in a post-Basel II regulatory environment. *European Journal of Operational Research*, 257 (3). pp. 1030-1044. ISSN 0377-2217

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# Market Risk Management in a Post-Basel II Regulatory Environment

Mikica Drenovak<sup>a</sup>, Vladimir Ranković<sup>a</sup>, Miloš Ivanović<sup>b</sup> Branko Urošević<sup>c</sup> and Ranko Jelic<sup>d</sup>

<sup>a</sup> Faculty of Economics, University of Kragujevac, Djure Pucara 3, 34000 Kragujevac, Serbia;

<sup>b</sup> Faculty of Science, University of Kragujevac, Radoja Domanovica 12, 34000 Kragujevac, Serbia

<sup>c</sup> Faculty of Economics, University of Belgrade, Kamenicka 6, 11000 Beograd, Serbia; National Bank of Serbia, Kralja Petra 12, 11000 Beograd, Serbia; CESifo, Munich, Germany.

<sup>d</sup> School of Business, Management and Economics, University of Sussex, Sussex House, Falmer, Brighton, BN1 9RH UK; Corresponding author: Ranko Jelic; phone: +44(0)1273872597; fax: +44(0)1273876677 ; email: [r.jelic@sussex.ac.uk](mailto:r.jelic@sussex.ac.uk).

## Abstract

We propose a novel method of Mean-Capital Requirement portfolio optimization. The optimization is performed using a parallel framework for optimization based on the Nondominated Sorting Genetic Algorithm II. Capital requirements for market risk include an additional stress component introduced by the recent Basel 2.5 regulation. Our optimization with the Basel 2.5 formula in the objective function produces superior results to those of the old (Basel II) formula in stress scenarios in which the correlations of asset returns change considerably. These improvements are achieved at the expense of reduced cardinality of Pareto-optimal portfolios. This reduced cardinality (and thus portfolio diversification) in periods of relatively low market volatility may have unintended consequences for banks' risk exposure.

**KEY WORDS:** *Finance, Market Risk, Basel 2.5, GARCH, NSGA-II*

## 1. Introduction

### 1.1. Capital requirements under Basel 2.5

The Basel II regulatory framework establishes the capital requirements (*CR*) for banks' exposure to market, credit and operational risks in terms of Value at Risk (*VaR*) estimations (BIS, 2006). *VaR* is a quantile measure of risk that is defined as an estimate of the maximum portfolio loss for a given holding period and a pre-set significance level. Banks are allowed to develop their own internal *VaR* models. These models are subject to supervisory approval based on standardized backtesting procedures. In the aftermath of the global financial crisis, the Basel Committee on Banking Supervision has identified the undercapitalization of banks' trading books and the pro-cyclicality of capital charges for market risk as the key weaknesses of the Basel II regulation. In response to the crisis, the Committee adopted the so-called Basel 2.5 regulation (BIS, 2009). Pursuant to the Basel 2.5 regulation, the *CR* required for market risk, calculated at day  $T$  to be held on day  $T+1$ , is determined as a sum of two components: *Regulatory VaR* (according to BIS, 2006) and *Regulatory Stressed VaR* (the additional capital charge mandated by BIS, 2009):

$$CR_{T+1} = \text{Regulatory VaR} + \text{Regulatory Stressed VaR} = \\ = \max(VaR_{h,T+1}^\alpha, (3+k) \times \frac{1}{60} \sum_{i=0}^{59} VaR_{h,T-i}^\alpha) + \max(SVaR_{h,T+1}^\alpha, (3+k) \times \frac{1}{60} \sum_{i=0}^{59} SVaR_{h,T-i}^\alpha) \quad (1)$$

Equation (1) requires that *VaR* estimates be made for a holding period  $h$  of 10 working days at a significance level  $\alpha$  of 1%. The value of the penalty parameter  $k$  in Equation (1) is based on the total number of *VaR* violations in the backtesting sample of the previous 250 days. Namely, for each day  $t$ ,  $t = T-249, T-248, \dots, T-1, T$ , the 1-day-ahead bank trading portfolio *VaR* estimated on day  $t-1$ ,  $VaR_t$ , is compared to the realized portfolio return at day  $t$ ,  $r_t$ . The number of *VaR* violations corresponds to the number of times that the realized portfolio loss (negative return) exceeds the loss predicted by the 1-day-ahead portfolio *VaR* estimate, that is when  $r_t < -VaR_t$ . Regulators use the number of violations as a proxy for the quality of the *VaR* modelling. Based on the number of *VaR* violations, the penalty parameter  $k$  can take discrete values between 0 and 1 and is proportional to the number of violations (see Table 1). An internal model is rejected if the number of violations is greater than or equal to 10.

**Table 1.** Value of the penalty parameter corresponding to the results of the backtesting procedure

Number of <i>VaR</i> violations	0-4	5	6	7	8	9	≥ 10
$k$	0	0.4	0.5	0.65	0.75	0.85	1

From Equation (1) and Table 1, it follows that the lowest possible *VaR* values might not necessarily lead to the lowest possible regulatory capital charge. Specifically, *Regulatory VaR* is determined as the maximum of the current 1-day-ahead *VaR* estimate and the average of the 1-day-ahead *VaR* estimates over the last 60 business days multiplied by the penalty factor  $3+k$ . Most of the time, the latter term dominates. Thus, a higher number of violations typically leads to a higher capital charge.

The expression for *Regulatory Stressed VaR* has the same structure as *Regulatory VaR* except that *VaR* estimates are replaced with their stressed counterparts, *SVaR*. The estimates for *SVaR* are determined for the same portfolio and in exactly the same way as *VaR* estimates, but assuming that the relevant market factors experienced stress during the most recent 250 days. Note that the value of  $k$  obtained from the backtesting of the original *VaR* estimates is also used in the *Regulatory Stressed VaR* expression.

It is important to note that the regulation requires banks to apply *CR* formula to a portfolio that they hold on the calculation date (the so-called actual portfolio). The actual portfolio framework (*APF*) implies that asset holdings on the date of the *CR* calculation, not the assets' portfolio weights (i.e., the fractions of portfolio value invested in each individual asset), are held fixed over the backtesting period. Therefore, when determining the value of the *CR*, it is crucial that not only all 1-day-ahead *VaR* estimates but also the penalty factor are based on the time series of returns of the actual portfolio. Fixing portfolio weights during the backtesting sample of 250 days would simplify calculations but would not be in line with the Basel regulation.

## 1.2. Contribution of the paper

In this paper, we propose a novel *Mean-CR* portfolio optimization approach in which the *CR* for market risk is calculated strictly in accordance with the Basel 2.5 regulation (Equation (1)). The *VaR* calculations are based on daily returns and 10-day *VaR* is obtained as the 1-day *VaR* multiplied by the square root of 10 (this approach is explicitly allowed by the regulation). We assume that conditional (time varying) variance of portfolio returns follows a univariate Generalized Autoregressive Conditional Heteroskedasticity (*GARCH* (1, 1)) process (see Bollerslev, 1986), whereas portfolio returns, standardized by conditional volatility, follow Student's  $t$  distribution. This model is referred to as the univariate *GARCH VaR* model. The univariate model of conditional variance efficiently captures the conditional variance of

portfolio returns and allows us to implement the backtesting procedure in accordance with the Basel 2.5 regulation.

*Mean-CR* optimization is subject to two key challenges: i) the use of the *APF* approach; and ii) the fact that the backtesting penalty parameter is a discrete function of *VaR* violations. As a result, the optimization problem that we solve is a highly complex, non-differentiable and non-convex. We manage this complexity by employing the Nondominated Sorting Genetic Algorithm II (*NSGA-II*) (Deb et al., 2000; Deb et al., 2002). When univariate *GARCH* model is used to estimate *VaR*, *CR* calculation for single portfolio becomes very time consuming. Thus, the *NSGA-II* algorithm cannot be executed on a single processor within a reasonable time frame. To address this problem, we employ a parallel framework for optimization based on the genetic algorithm developed by Ivanovic et al. (2015).

In the empirical section, we test our approach using an opportunity set of 40 constituents of the Standard and Poor's 100 index (*S&P 100*). Specifically, we examine two samples representing high- and low-volatility market environments and include comparison of four stress scenarios. We show that *Mean-CR* optimal portfolios, presented in the *Mean-CR* plane, outperform *Mean-Regulatory VaR* optimal portfolios only when the original correlation of asset returns significantly differs from the correlations imposed by the stress scenarios. This is the case for two of the four stress scenarios that we employ. The improvements are particularly pronounced in a low-volatility environment. However, they are achieved at the expense of the reduced cardinality of Pareto-optimal portfolios. For this reason, the additional capital charge in the Basel 2.5 formula might result in less diversified optimal portfolios, especially during periods of relatively low market volatility. We see this as an unintended consequence of the current regulation. In addition, we show that although the *Mean-VaR* optimization is much simpler, generally does not lead to near optimal *Mean-CR* trade-offs.

### **1.3. Relation to the literature**

Santos et al. (2012) is the only previous study on portfolio optimization that uses *CR* as the objective function. Those authors propose an analytical model to determine optimal portfolios with minimum *CR*. They convexify *CR* and impose an ad-hoc (exogenous) limitation on the number of *VaR* violations. In determining *CR*, the authors use conditional multivariate *GARCH VaR* model. Unlike our univariate *GARCH VaR* approach, their model implicitly assumes that assets' portfolio weights, rather than asset holdings, are fixed over the observed

time period. The two *GARCH VaR* modelling approaches are compared in detail in Ranković et al. (2016). That paper pioneers the use of *APF* in *Mean-VaR* portfolio optimization, showing that within the *APF* approach, the use of univariate *GARCH VaR* outperforms its multivariate *GARCH VaR* counterpart. We significantly extend Ranković et al. (2016) work by applying their approach to construct the *Mean-CR* Pareto-optimal front.

In addition to the three hypothetical scenarios considered in Santos et al. (2012), we also consider a historical stress scenario. This historical scenario explores what would have happened to our portfolio if adverse historical conditions were to re-occur. The use of the historical stress scenario is consistent with regulatory changes suggesting that *CR* should be calibrated to a period of significant market stress in both internal and standard regulatory models (BIS, 2009). Both historical and hypothetical stress scenarios are frequently employed in banking practice (Alexander, 2008a). The advantage of historical scenarios is that they consist of credible assumptions (i.e., events that have actually occurred in the past). In contrast, hypothetical scenarios allow us to make our own assumptions about the future based on both current market conditions and the specifics of our portfolio.

The real-world optimization problems in finance usually include multiple conflicting objectives, non-differentiable objective functions, large and non-convex solution spaces, complex constraints etc. which are not solvable using traditional analytical techniques. Recently, metaheuristics, such as multi-objective evolutionary algorithms (*MOEAs*) have become very popular in solving complex portfolio optimization problems (e.g. see Branke et al., 2009).<sup>1</sup> *MOEAs* have the ability to generate the entire Pareto-optimal front in a single run. However, they require numerous solution evaluations. An important characteristic of *MOEAs* is that the evaluations of solutions within a single generation are independent and therefore suitable for parallel execution. Ivanovic et al. (2015) propose a parallel framework for optimization based on a genetic algorithm (*WoBinGO*). To solve multi-objective problems, those authors employ *NSGA-II*, showing that their framework provides a better execution time of two orders of magnitude for solving computationally extensive problems than serial execution.

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<sup>1</sup> A detailed overview of *MOEA* applications in finance can be found in Schlottmann and Seese (2004), Tapia and Coello (2007), Metaxiotis and Liagkouras (2012) and Ponsich et al. (2013).

The remainder of this paper proceeds as follows. Section 2 presents the univariate *VaR* model. The portfolio optimization problem is formally introduced in Section 3. In Section 4, we discuss the optimization methodology. In Section 5, we present data and sample characteristics. Section 6 reports on empirical results and analyses the impact of various stress scenarios and robustness checks. We conclude the paper in Section 7.

## 2. *VaR* model

Based on the empirical characteristics, financial asset returns are often presented as a function of first two conditional moments of distribution  $r_t = \mu_t + \sigma_t z_t$ , where  $z_t$  is the innovation term of the process assumed to be independently and identically distributed. In estimating *VaR*, we assume that the conditional mean of daily returns  $\mu_t$  is dominated by the conditional volatility of returns  $\sigma_t$  ( $\mu_t \ll \sigma_t$ ). This implies that daily portfolio returns can be approximated as  $r_t \approx \sigma_t z_t$  (see Alexander, 2008a; Christofferson, 2012; Pritsker, 2006). In addition, we assume that innovations  $z_t$  follow a standardized Student's  $t$  distribution with conditional variance following the most popular univariate *GARCH* (1, 1) model (see Bollerslev, 1986):

$$\sigma_{t+1}^2 = \omega + \theta r_t^2 + \beta \sigma_t^2 \quad (2)$$

Here,  $\theta, \beta > 0$  and  $\theta + \beta < 1$ . The Student's  $t$  distribution can explain heavy-tailed distributions of various degrees (see Christoffersen, 2012; Huisman et al., 1998) thus allowing us as to account for conditional nonnormality in portfolio returns. The *GARCH* model efficiently captures the volatility clustering that is often present in empirical returns.<sup>2</sup> To estimate *GARCH VaR* for the portfolio under consideration, we first fit the univariate *GARCH* (1, 1) model on a time series of 1,000 daily portfolio returns (see Equation (2)). Next, we multiply the  $\alpha$ -quantile of the fitted standardized return distribution by the current 1-day-ahead conditional volatility estimate. Our 1-day-ahead *VaR* estimate is calculated using the following formula:

$$VaR_{t+1}^\alpha = -\left(\omega + \theta r_t^2 + \beta \sigma_t^2\right)^{1/2} t_\alpha^{-1}(d) \quad (3)$$

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<sup>2</sup> For more on the superiority of the *GARCH* model for *VaR* estimation, see Alexander (2008a), Alexander and Sheedy (2008), Berkowitz and O'Brien (2002), Hull and White (1998), and Pritsker (2006), among others.

Here,  $d$  is the number of degrees of freedom of the estimated Student's  $t$  distribution for the given portfolio, and  $t_{\alpha}^{-1}(d)$  is the  $\alpha$ -quantile of the standardized Student's  $t$  distribution with  $d$  degrees of freedom. Note that in the univariate *GARCH VaR* model, the distribution of each portfolio's returns has its own number of degrees of freedom. The number of degrees of freedom is estimated jointly with the corresponding *GARCH*  $(1, 1)$  parameters.

### 3. Actual portfolio *Mean-CR* optimization problem

A bi-objective portfolio optimization problem typically aims to minimize portfolio risk and maximize expected portfolio return. The optimization is subject to constraints that define a set of feasible portfolios. In this paper, we minimize  $CR$  (given by Equation (1)) and maximize the expected return on the portfolio. Specifically, we are attempting to solve the following problem:

$$\min_{\mathbf{w}_T} CR_{T+1} \quad (4)$$

$$\max_{\mathbf{w}_T} E(r) \quad (5)$$

$$\text{subject to} \quad \sum_{i=1}^N w_{i,T} = 1 \quad (6)$$

$$0 \leq w_{i,T} \leq 1, \quad i = 1, \dots, N \quad (7)$$

Here,  $T$  denotes the optimization date;  $CR_{T+1}$  are the capital requirements for market risk, calculated at day  $T$ , to be held on day  $T+1$  (Equation (1));  $E(r)$  is the expected return on portfolio at day  $T$ , defined as the sample mean portfolio return over the observed time horizon of 1,000 days; and  $\mathbf{w}_T$  is the vector of portfolio weights  $w_{i,T}$  at day  $T$ . Decision variables are represented by vector  $\mathbf{w}_T$ . Note that the portfolio of assets is typically defined by weights because such a representation is independent of budget level. The first constraint (Equation (6)) ensures that weights sum up to 1. Expression (7) ensures the non-negativity of each investment consistent with the absence of short sales.

When estimating  $CR$ , we follow the *APF* approach introduced in Rankovic et al. (2016). Let  $n_i$  denote holdings of asset  $i$  at date  $T$ . We can express  $n_i$  via corresponding portfolio weights  $w_{i,T}$ :

$$n_i = \frac{w_{i,T} V_{p,T}}{P_{i,T}} \quad (8)$$



Here  $V_{p,T}$  and  $P_{i,T}$  are the dollar portfolio value and the price of asset  $i$  at time  $T$ , respectively. To obtain returns on the actual portfolio, we hold fixed asset holdings  $n_i$  over time. Therefore, the return on the actual portfolio at time  $t \leq T$  is given by the following expression:

$$r_t = \frac{V_{p,t}}{V_{p,t-1}} - 1 = \frac{\sum_{i=1}^N n_i P_{i,t}}{\sum_{i=1}^N n_i P_{i,t-1}} - 1 \quad (9)$$

Substituting expression (8) into (9), return on the actual portfolio at time  $t$  can be expressed in terms of portfolio weights at time  $T$ :

$$r_t = \frac{\sum_{i=1}^N \frac{w_{i,T} V_{p,T}}{P_{i,T}} P_{i,t}}{\sum_{i=1}^N \frac{w_{i,T} V_{p,T}}{P_{i,T}} P_{i,t-1}} - 1 = \frac{\sum_{i=1}^N \frac{w_{i,T}}{P_{i,T}} P_{i,t}}{\sum_{i=1}^N \frac{w_{i,T}}{P_{i,T}} P_{i,t-1}} - 1 \quad (10)$$

Equations (8) – (10) imply that in the *APF* approach portfolio weights change over time while holdings remain fixed. For a given actual portfolio (i.e., for a given vector  $\mathbf{w}_T$ ), the time series of portfolio returns are obtained using Equation (10), whereas *VaR* and *SVaR* estimations are calculated using Equation (3). In the case of *Regulatory VaR*, in Equation (10) we use realized market prices during the backtesting period. In the case of *Regulatory Stressed VaR*, we use the market prices under four alternative stress scenarios. We discuss specification of the alternative stress scenarios used in the empirical analysis in Section 6.2.

#### 4. Mean-CR optimization based on evolutionary algorithm (EA)

EAs start with a set of randomly generated candidate solutions, referred to as a population. In each of the iterations (generations), a set of new candidate solutions (offspring solutions) is generated by applying the evolutionary processes of: selection, crossover and mutation. As these procedures are repeated, their solutions evolve and improve in terms of the chosen objectives. To evaluate each candidate solution, we first generate a time series of returns on the actual portfolio and stressed time series of realized portfolio returns by applying Equation (10). We then calculate *Regulatory VaR* by performing the backtesting procedure on the time series of returns on the actual portfolio and determine the 1-day-ahead *VaR*, the last 60-day *VaR* average and penalty parameter  $k$ . Because we use the univariate *GARCH* (1, 1) model for

$VaR$  estimation, the backtesting procedure for every single candidate portfolio requires an estimation of 250 sets of  $GARCH$  parameters. We used the ‘rugarch’ package (Ghalanos, 2014) within the software R (R Core Team, 2014).  **$GARCH$**  model parameters are determined, using maximum likelihood ( $ML$ ) estimation, with ‘ugarchfit’ method. Estimation of a single set of  $GARCH$  parameters is based on a time series of 1,000 portfolio returns. It is worth noting that using the long time series of portfolio returns increases numerical stability of  $ML$  estimation. For the 1-day-ahead estimation of conditional volatility, we used the ‘ugarchforecast’ method.

We use a rolling window of 1,000 returns for the backtesting period of 250 days (see Figure 1). Here,  $T$  is the optimization date, whereas  $t$  refers to an arbitrary date within the backtesting period. Notably, the number of  $VaR$  violations is determined endogenously (within the backtesting procedure) as a part of the objective function evaluation.

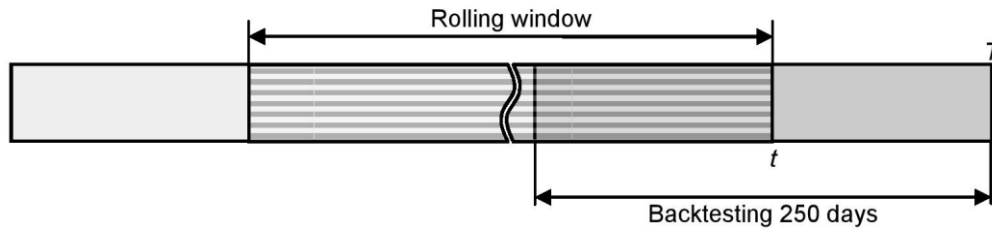


Figure 1. *Regulatory VaR* calculation

Next, we calculate *Regulatory Stressed VaR*, which is based on the same penalty parameter  $k$  obtained in the estimation of *Regulatory VaR*. Thus, we need to calculate the  $SVaR$  average of the last 60 days. This requires the estimation of 60 additional sets of  $GARCH$  parameters. Here, we use a rolling window of 1,000 returns for a period of 60 days. The latest 250 returns of the sample ending at the optimization date  $T$  are stressed returns (See Figure 2). Here,  $t$  is an arbitrary date within the period of the last 60 days. In sum, to calculate  $CR$  for a single candidate portfolio we need 310  $ML$  estimations, 250 estimations for *Regulatory VaR* and 60 estimations for *Regulatory Stressed VaR*.

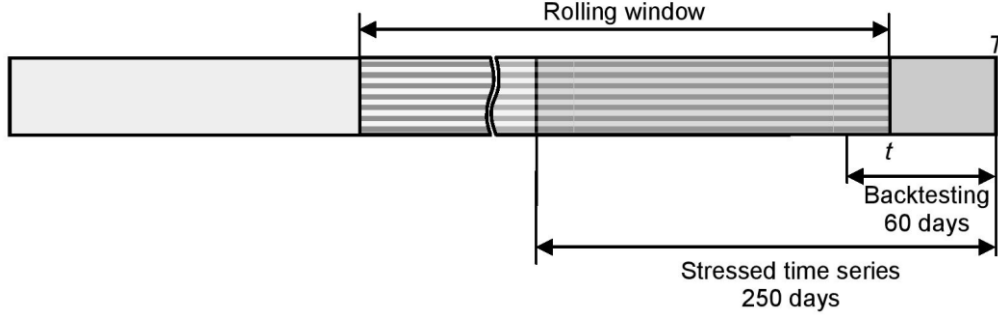


Figure 2. *Regulatory Stressed VaR calculation*

*NSGA-II* is executed within *WoBinGO* parallel framework on a cluster of 100 dedicated processors. In the preparatory phase, the main *WoBinGO* component (*master*) generates a data file (*timeSeries.csv*) with a time series of assets' prices, a data file (*stressedTimeSeries.csv*) with a stressed time series of assets' prices and an R script file (*Script.R*) containing the sequence of commands for *CR* calculation. The files are then automatically uploaded to a file system shared among all cluster nodes.

In the execution phase, the *master* executes the main evolutionary loop (i.e., the loop over generations). For each individual solution within a generation, *WoBinGO* generates a file with portfolio weights (*weights.csv*) and uploads it to the first available processor. Next, it invokes R, which executes *Script.R* created in the preparatory phase. During the execution of the script file, R uses the data file *timeSeries.csv*, the data file *stressedTimeSeries.csv* and the solution portfolio weights file *weights.csv*, generating the time series of returns and stressed time series of returns on the actual portfolio (applying Equation (8)). The *CR* is, then, calculated applying Equation (1), and the result (*CR* and *Mean*) is returned back to the main evolutionary loop and assigned to the corresponding solution. When all solutions from a single generation are evaluated, the *master* proceeds with the evolutionary algorithm.

The implementation of *NSGA-II* requires settings for the solution representation, the population size, the crossover and mutation probabilities and the termination condition. Driven by the considered optimization problem, we define solution as a non-negative real-valued vector of portfolio weights. The population size is set to 100.

To breed the offspring population, a uniform crossover operator is employed. Portfolios from the current population are randomly selected and recombined with a predefined crossover

probability that equals 1. The recombination implies that every allele (i.e., individual asset weight) is exchanged between the pair of parent solutions with a certain probability, which is known as the swapping probability (see Sastry et al., 2005). In accordance with the previous literature, we set the swapping probability to 0.5.

For the mutation process, we apply a uniform mutation operator. The operator implies that each allele is selected with a predefined mutation probability and replaced with a realization of a random variable, which is uniformly distributed in the range defined by the lower and upper domain bounds. We set the mutation probability to 0.05. The selected crossover and mutation operators satisfy the constraint defined by Equation (7) for each offspring. However, these operators do not ensure satisfaction of the budget constraint (Equation (6)). Thus, we normalize each of the offspring solutions. We do that in the standard way by dividing each weight by sum of all weights.

To reduce the execution time, we introduce a termination condition based on hypervolume measure (see Zitzler et al., 2003). The hypervolume quantifies the volume of the objective solution space dominated by an approximation set. For optimization problems with two objectives, it quantifies the area of the objective space dominated by the approximation set. Here, the area is bounded by a predefined reference point defined by the minimum return and maximum *CR* achieved in the current generation. Therefore, the termination condition is defined in terms of the relative increase of the hypervolume. If the relative increase of the hypervolume is not greater than  $5 \times 10^{-4}$  in 10 successive generations, the algorithm stops. The maximum number of generations is set to 100.

## 5. Data and sample characteristics

For the sake of easier comparison, we use the same data sample as in Ranković et al. (2016). Specifically, we use 40 constituent stocks of the *S&P 100* with the highest market capitalization (as of September 6, 2013) and with daily price observations available from January 2007. To determine the maximum and minimum volatility dates, the 1-day-ahead daily volatility of the *S&P 100* is estimated using the rolling estimation period from January 4, 2012 to September 6, 2013. Volatilities are estimated using the *GARCH (1, 1)* model. Standardized returns are assumed to have a standardized Student's *t* distribution. *GARCH* volatility estimations of the *S&P 100* index are based on the rolling window of 1,000 daily

returns. Specifically, a maximum volatility of 20.5% in annual terms is found on June 29, 2012 whereas a minimum volatility of 9.3% is determined on July 31, 2013. We use these dates to create two samples of time series with 1,251 daily prices. One thousand, two hundred and fifty-one prices give us 1,250 returns (1,000 returns for *GARCH* parameters estimation and 250 returns for backtesting). One sample ends on the maximum volatility date (we refer to this sample as the high-volatility sample), whereas the other ends on the minimum volatility date (we refer to this sample as the low-volatility sample).

## 6. Results

We perform two sets of empirical experiments. First, we apply the proposed *NSGA-II* algorithm to construct two benchmark solution sets based on simpler optimization problems: the *Mean-VaR* and *Mean-Regulatory VaR* Pareto-optimal sets of portfolios. We present these two sets of solutions in the *Mean-Regulatory VaR* coordinates. When performing the *Mean-Regulatory VaR* optimization, we use only the first part of Equation (1) and therefore do not address stressed time series. In addition, we use the *Mean-VaR* Pareto-optimal set as the initial population. This reduces (to an extent) the computational complexity but still requires parallel computing (note that the *APF Mean-VaR* optimization can easily be performed on a single processor). For these two benchmark sets, we then calculate the corresponding *CR* using the Basel 2.5 formula (Equation (1)), utilizing four proposed stress scenarios. We adopt these values as benchmarks for the *Mean-CR* optimized portfolios. Finally, we generate *Mean-CR* Pareto-optimal fronts and compare the *Mean-VaR*, *Mean-Regulatory VaR* and *Mean-CR* Pareto-optimal sets in the *Mean-CR* coordinates for every stress scenario.

### 6.1. Basel II capital requirements: *Mean-Regulatory VaR* optimization

In this subsection, we present the results of the *Mean-Regulatory VaR* optimization and compare them, in the *Mean-Regulatory VaR* plane, with our benchmark *Mean-VaR* Pareto-optimal sets for the low- (Figure 3) and high- (Figure 4) volatility samples.<sup>3</sup>

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<sup>3</sup> We annualize expected return assuming 252 days per annum.

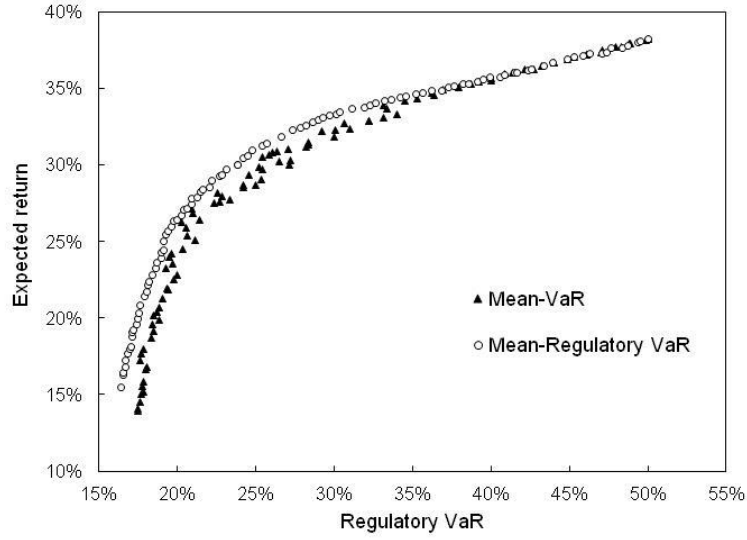


Figure 3. *Mean-Regulatory VaR* performance of optimized portfolios for the low-volatility sample

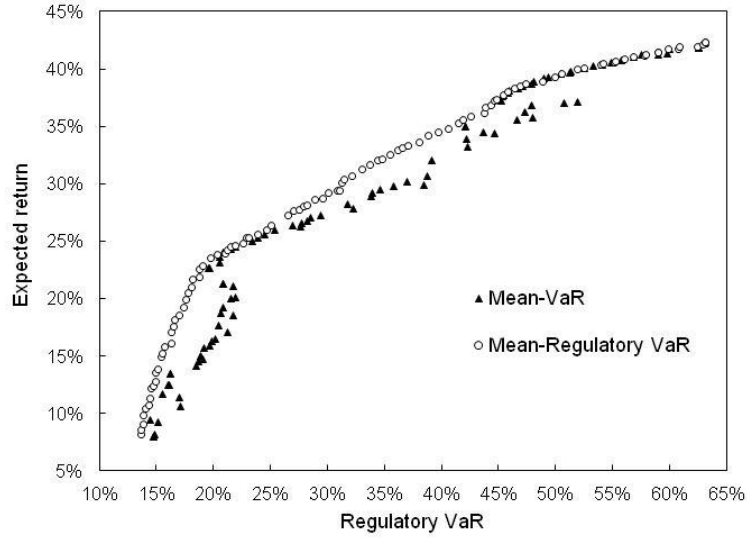


Figure 4 *Mean-Regulatory VaR* performance of optimized portfolios for the high-volatility sample

Differences in the two Pareto-optimal sets are more evident in the high-volatility sample (Figure 4), particularly for portfolios with lower expected returns, which are normally more diversified portfolios. It is worth recalling here that the cardinality of optimal portfolios always increases with a decrease in the expected return. Thus, portfolios with very high expected returns consist of only a few assets. By construction, the portfolio with the highest return consists of a single asset.

In Table 2, we present the number of *VaR* violations for *Mean-VaR* and *Mean-Regulatory VaR* optimized portfolios. *Mean-Regulatory VaR* optimization provides better trade-offs than

*Mean-VaR* optimization. Notably, the algorithm finds it optimal to always keep the number of violations within the no-penalty zone. In contrast, in the high-volatility regime, twenty (out of one hundred) *Mean-VaR* optimized portfolios lead to more than four violations. The maximum number of violations was seven. Thus, the lowest possible *VaR* estimations do not necessarily lead to the lowest possible *Regulatory VaR* because of the penalties associated with the number of *VaR* violations.

**Table 2.** Number of violations – Basel II

Sample	Optimization criterion	Number of violations			
		Mean	Maximum	Minimum	More than 4
Low volatility	<i>VaR</i>	1.03	2	0	0
	<i>Regulatory VaR</i>	1.54	4	0	0
High volatility	<i>VaR</i>	3.26	7	2	20
	<i>Regulatory VaR</i>	2.93	4	2	0

As expected, the average number of violations for both the *Mean-VaR* and the *Mean-Regulatory VaR* optimized portfolios is significantly higher in the high-volatility regime. We conclude that increased optimization complexity, implied by moving from *Mean-VaR* to *Mean-Regulatory VaR* is generally warranted.

## 6.2. Basel 2.5 capital requirements (CR) in four stress scenarios

Now we address the central issue of this paper, that is, how to minimize capital charge *CR* (see Equation 1) while maximizing the expected portfolio returns. In doing so, we construct a new type of Pareto-optimal front. This is an important practical issue for all institutions that follow Basel 2.5 rules. With the addition of the *Regulatory Stressed VaR* term, the overall level of the required regulatory capital charge has been substantially increased with respect to the level determined by the original Basel II regulation. In addition, different stress scenarios could have different impacts on the optimal *Mean-CR* trade-off. When performing the *Mean-CR* optimization, we use the *Mean-Regulatory VaR* Pareto-optimal set as the initial population.

We solve the *Mean-CR* optimization problem and compare the results with the *Mean-Regulatory VaR* and *Mean-VaR* Pareto-optimal sets in the *Mean-CR* plane. This gives us an idea of how close we can get to the *Mean-CR* Pareto-optimal front when utilizing simpler *Mean-VaR* and *Mean-Regulatory VaR* optimization procedures.

When calculating the *Regulatory Stressed VaR*, we apply one historical stress test and three hypothetical tests. Following the regulation, the last 250 prices of the original time series of

asset prices must be replaced by stressed prices. Thus, for each stress test we generate stressed time series of asset prices so that their last 250 returns correspond to a chosen stress scenario. The rest of the time series of prices remains unchanged. Next, we utilize Equation (10) to calculate the actual portfolio returns corresponding to such a stressed price series. The results for all stressed scenarios are presented for both the low- and high-volatility samples.

To perform a historical stress test, we begin by calculating the moving standard deviation of returns on the *S&P100* index (used here as a proxy for the market portfolio) for 60-day time intervals for the period between January 16, 2008 and January 4, 2010. We use 60-day time intervals for the volatility estimation because *Regulatory Stressed VaR* is based on average *VaR* for a 60-day period. We find that the maximum volatility of 4.55% is recorded for the 60-day period that ended on December 8, 2008. For this reason, to obtain the stressed time series, we replace the original asset prices for the last 250 days with the prices implied by historical returns for the period December 12, 2007 to December 8, 2008.

In addition, we consider three hypothetical stress scenarios consistent with the three scenarios examined in Santos et al. (2012). Under the first scenario (*HSS1*), we apply a uniform haircut of 20% to the last 250 asset returns. For each portfolio asset  $i$ , the stressed return is defined as follows:  $r_{i,t}^{stressed} = r_{i,t} - 0.2 \text{Mean}(r_i)$ , where  $t=T-249, T-248, \dots, T$ . Here,  $r_{i,t}^{stressed}$  is a stressed return of asset  $i$  at time  $t$ ,  $r_{i,t}$  is the original return of asset  $i$  at time  $t$  and  $\text{Mean}(r_i)$  is the mean of the last 250 returns of asset  $i$ . We generate stressed time series of assets' prices by adjusting the original time series. Formally,  $P_{i,t}^{stressed} = P_{i,t-1}^{stressed} (1 + r_{i,t}^{stressed})$ ,  $t=T-249, T-248, \dots, T$  and  $P_{i,T-250}^{stressed} = P_{i,T-250}$ . Here,  $P_{i,t}$  and  $P_{i,t}^{stressed}$  are the original and stressed prices of asset  $i$  at time  $t$ , respectively.

Under the second scenario (*HSS2*), we start with the stressed returns obtained under *HSS1* and double their volatility. Note that a covariance matrix of asset returns  $\mathbf{V}$  can be decomposed as  $\mathbf{V}=\mathbf{D}\mathbf{C}\mathbf{D}$ . Here,  $\mathbf{D}$  is the diagonal matrix of the standard deviations of asset returns and  $\mathbf{C}$  is the corresponding correlation matrix. We construct  $\mathbf{D}^{stressed}$ , a diagonal matrix at date  $T$ , by placing stressed volatilities  $\sigma_{i,T}^{stressed} = 2\sigma_{i,T}$  on the diagonal. Here  $\sigma_{i,T}$  is the sample standard deviation of the  $i$ -th asset calculated using the last 250 returns. The stressed covariance matrix is  $\mathbf{V}^{stressed}=\mathbf{D}^{stressed}\mathbf{C}\mathbf{D}^{stressed}$ . To obtain stressed returns for  $n$  assets from the opportunity set we employ Cholesky matrix decomposition. We decompose both the original and stressed covariance matrices as follows:  $\mathbf{V}=\mathbf{Q}\mathbf{Q}'$  and  $\mathbf{V}^{stressed}=\mathbf{Q}^{stressed}(\mathbf{Q}^{stressed})'$ , where  $\mathbf{Q}$  and  $\mathbf{Q}^{stressed}$  are lower triangular matrices. If  $\mathbf{R}$  is the returns matrix of dimensions  $250 \times n$  (in this matrix



columns represent time series of  $n$  assets entering the opportunity set), then the matrix of stress-adjusted historical returns  $\mathbf{R}^{stressed}$  is obtained as  $\mathbf{R}^{stressed} = \mathbf{R}(\mathbf{Q}^{stressed}\mathbf{Q}^{-1})'$  (See Alexander, 2008b). Stressed time series of asset prices are obtained from matrix  $\mathbf{R}^{stressed}$  by adjusting the original time series of prices in the same fashion as under *HSS1*.

In the third scenario (*HSS3*), we repeat the exercise from the previous scenario. In addition, we stress the correlation matrix so that  $\rho_{i,j}^{stressed} = \min(2\rho_{i,j}, 0.95)$ ,  $i \neq j$ . Here,  $\rho_{ij}$  and  $\rho_{i,j}^{stressed}$  are the original and stressed sample correlation coefficients between assets  $i$  and  $j$ , calculated using the last 250 returns. The stressed covariance matrix is now  $\mathbf{V}^{stressed} = \mathbf{D}^{stressed}\mathbf{C}^{stressed}\mathbf{D}^{stressed}$ . Here,  $\mathbf{D}^{stressed}$  is defined as in *HSS2*. We then generate the matrix of stressed historical returns  $\mathbf{R}^{stressed}$  and the corresponding stressed asset prices following the same procedure as in *HSS2*. In modifying the correlations, we need to ensure positive semi-definiteness of the stressed covariance matrix. If the obtained stressed covariance matrix is not positive semi-definite, we apply the algorithm for finding the nearest positive semi-definite covariance matrix based on the Frobenius norm (see Higham, 2002). To implement the algorithm we used the ‘nearPD’ method from ‘Matrix’ package in software R.

### 6.2.1. Historical stress scenario

Here we present the results of *Mean-CR* optimization for the historical stress scenario. For comparison, we also calculate *CR* for the *Mean-VaR* and *Mean-Regulatory VaR* optimized portfolios under the historical stress scenario. *Mean-CR* optimization leads to particularly large improvements in low-volatility sample vis-à-vis the benchmarks (Figure 5). In contrast, improvement vis-à-vis *Mean-Regulatory VaR* optimization is less pronounced in the high-volatility sample (Figure 6).

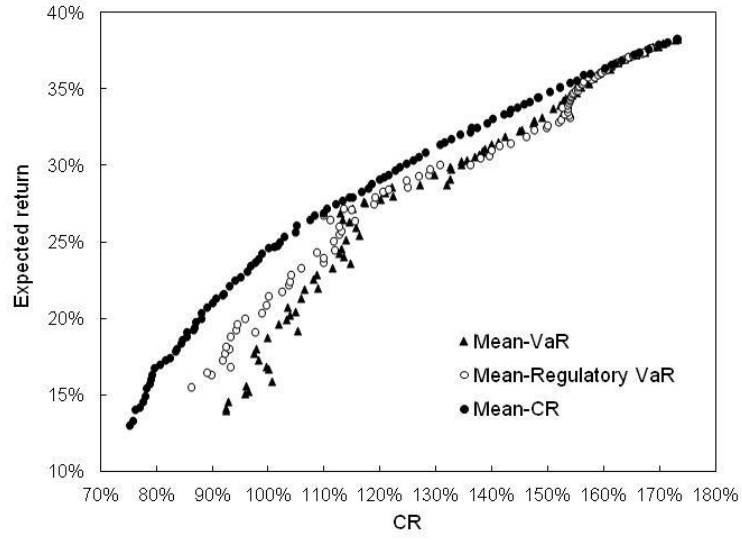


Figure 5. *Mean-CR* performance for historical stress scenario: low-volatility sample

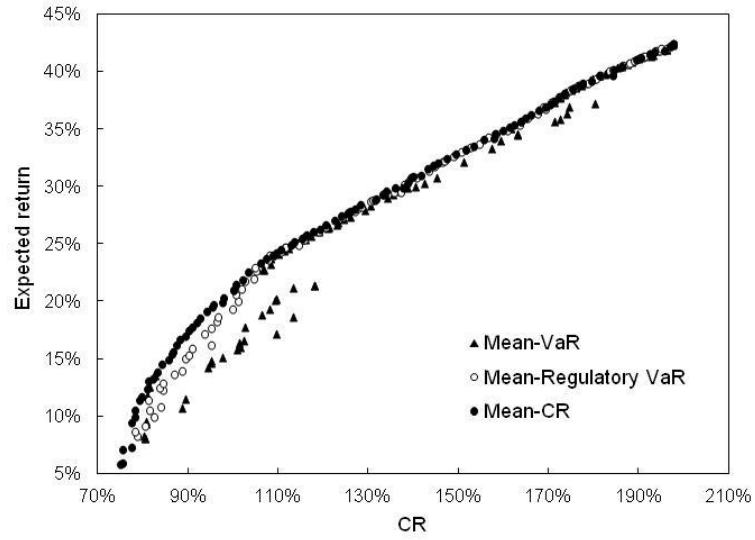


Figure 6. *Mean-CR* performance for historical stress scenario: high-volatility sample

Table 3 presents the number of *VaR* violations in the low- and high-volatility samples. *Mean-CR* optimization keeps the number of violations of optimal portfolios within the no-penalty zone. Consistent with the benchmarks, the average number of violations is higher in the high-volatility sample.

**Table 3.** Number of violations: Historical stress scenario

Sample	Optimization criterion	Number of violations			
		Mean	Maximum	Minimum	More than 4
Low volatility	<i>VaR</i>	1.03	2	0	0
	<i>Regulatory VaR</i>	1.54	4	0	0
	<i>CR</i>	1.92	3	0	0
High volatility	<i>VaR</i>	3.26	7	2	20
	<i>Regulatory VaR</i>	2.93	4	2	0
	<i>CR</i>	3.03	4	2	0

*Mean-VaR* optimization, although much simpler, performs remarkably well in the case of larger returns, especially for the high-volatility sample (see Figure 6). However, *Mean-VaR* optimization does not lead to the near optimal *Mean-CR* trade-offs for the entire Pareto front.

### 6.2.2. Hypothetical stress scenarios

Here, we present the results of *Mean-CR* optimization for the three hypothetical stress scenarios. The results presented in Figures 7 and 8 suggest that, under *HSS1*, *Mean-CR* optimization does not improve *Mean-CR* trade-offs compared to the *Mean-Regulatory VaR* optimal set. In that situation additional computational complexity is likely not warranted. This conclusion is supported by the results for the number of *VaR* violations in the low- and high-volatility samples for the three scenarios (see Table 4). The two models exhibited identical minimum and maximum numbers of violations for the two optimization dates and very similar average numbers of violations. Unsurprisingly, the *Mean-VaR* optimization solutions perform worse, particularly in the high-volatility sample.

**Table 4.** Number of violations: Hypothetical stress scenario scenarios *HSS1-HSS3*

Sample	Optimization criterion	Number of violations			
		Mean	Maximum	Minimum	More than 4
Low volatility	<i>VaR</i>	1.03	2	0	0
	<i>Regulatory VaR</i>	1.54	4	0	0
	<i>CR-HSS1</i>	1.48	4	0	0
	<i>CR-HSS2</i>	1.48	4	0	0
	<i>CR-HSS3</i>	1.31	2	0	0
High volatility	<i>VaR</i>	3.26	7	2	20
	<i>Regulatory VaR</i>	2.93	4	2	0
	<i>CR-HSS1</i>	2.98	4	2	0
	<i>CR-HSS2</i>	2.95	4	2	0
	<i>CR-HSS3</i>	2.88	4	2	0

Similarly, we observe no significant differences between the *Mean-Regulatory VaR* and *Mean-CR* optimized portfolios (Figures 9 and 10) when *HSS2* is applied. Here, the *Mean-VaR* benchmark lags behind even more significantly, particularly for the high-volatility sample.

In *HSS3*, the *Mean-CR* optimization provides a significant improvement with respect to the *Mean-Regulatory VaR* benchmark, particularly for the low-volatility sample (Figures 11 and 12). In that case, the impact of the *HSS3* stress scenario is qualitatively similar to the case of the historical stress scenario. In contrast to the historical scenario, however, an improvement, albeit smaller, is now recorded in the high-volatility sample.

Note that for the historical and all three hypothetical stress scenarios and in both volatility samples, *Mean-CR* optimization keeps the number of violations within the no-penalty zone (i.e., less than or equal to 4).

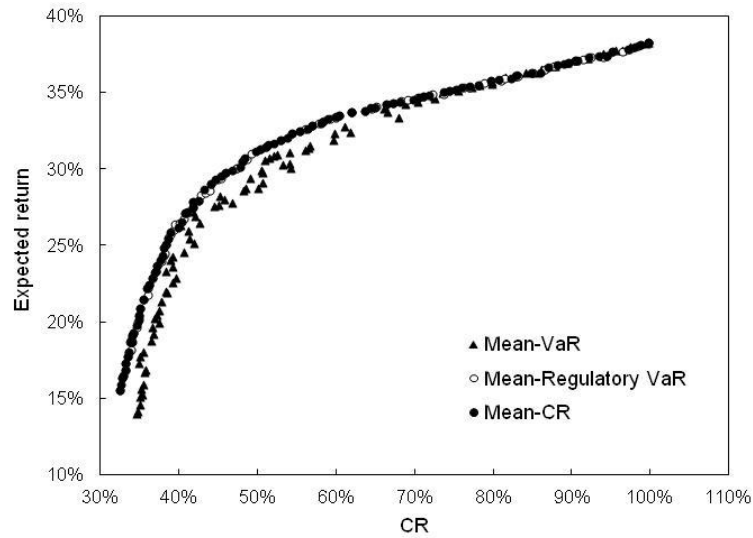


Figure 7. *Mean-CR* performance for *HSS1* stress scenario: low-volatility sample

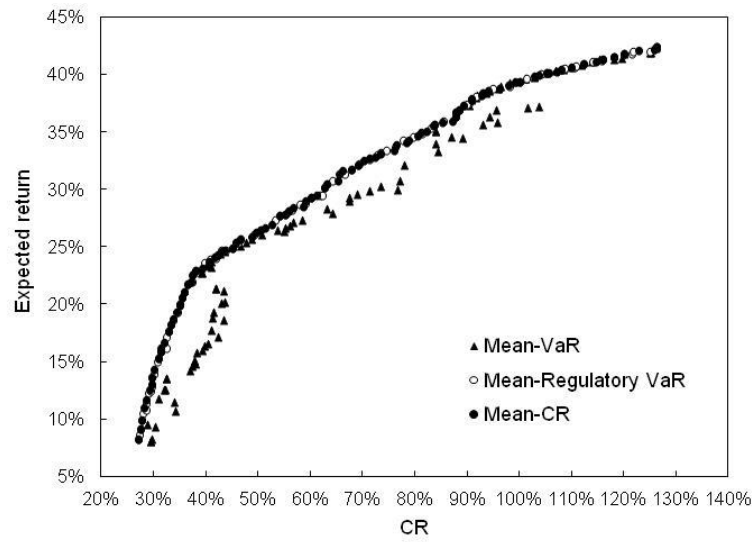


Figure 8. *Mean-CR* performance for *HSS1* stress scenario: high-volatility sample

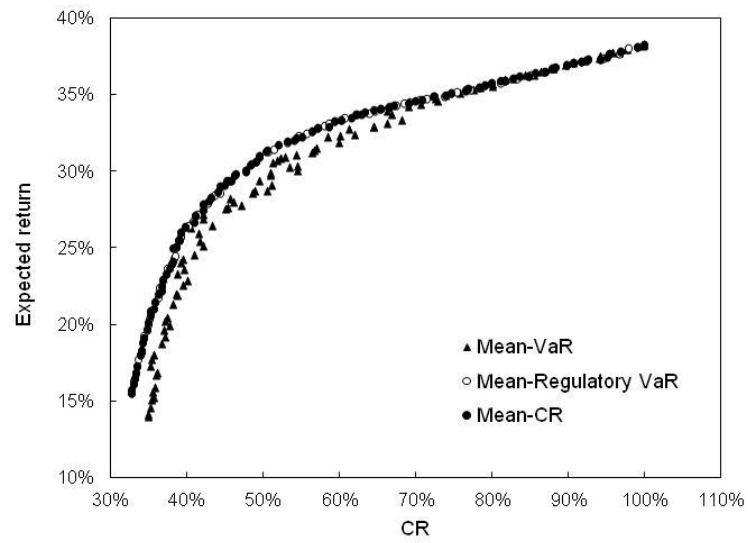


Figure 9. *Mean-CR* performance for *HSS2* stress scenario: low-volatility sample

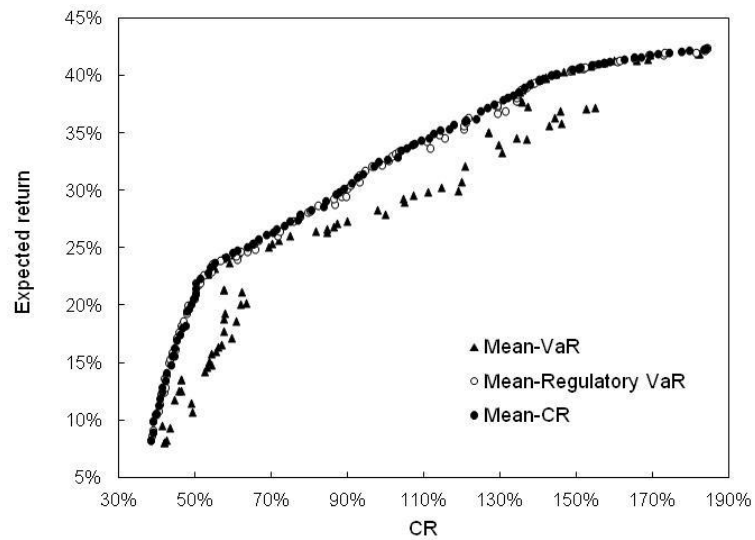


Figure 10. *Mean-CR* performance for *HSS2* stress scenario: high-volatility sample

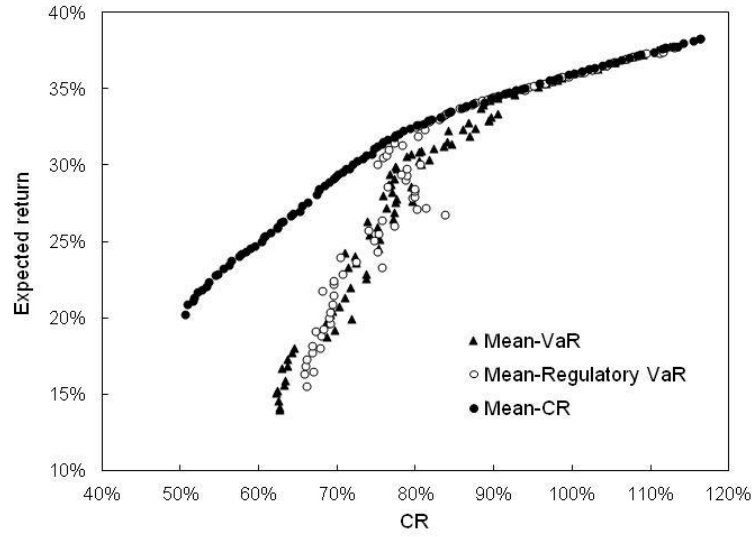


Figure 11. *Mean-CR* performance for *HSS3* stress scenario: low-volatility sample

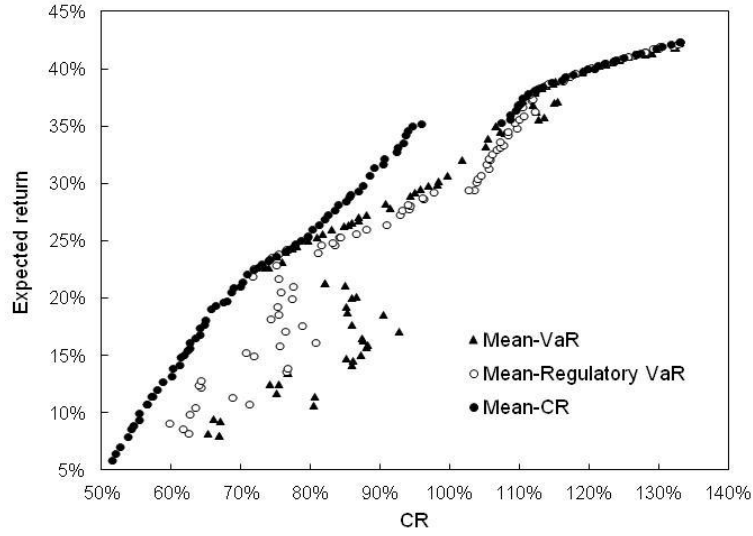


Figure 12. *Mean-CR* performance for *HSS3* stress scenario: high-volatility sample

### 6.3. Further analysis and robustness checks

In this subsection, we perform further analyses and robustness checks. Specifically, we: i) examine the relationship between the average change in correlation (imposed by stress tests) and corresponding differences in the *Mean-CR* trade-offs; ii) examine cardinality of the Pareto-optimal portfolios; and iii) repeat our *Mean-CR* optimization using an alternative opportunity set.

### 6.3.1. Mean-Regulatory VaR vs. Mean-CR optimization

To quantify differences in correlation matrices between the original and stressed returns, we utilize the average (mean) *Correlation difference*, calculated using the last 250 returns as follows:

$$Correlation\ difference = Abs\left(\frac{\rho_{i,j}^{stressed}}{\rho_{i,j}} - 1\right) \quad (11)$$

where  $\rho_{i,j}^{stressed}$  and  $\rho_{i,j}$  denote the stressed and the original correlations between assets  $i$  and  $j$ ,  $i \neq j$ , respectively. We also quantify the differences between the *Mean-CR* and *Mean-Regulatory VaR* optimal portfolios in terms of both *CR* and returns. First, we determine portfolios in the *Mean-Regulatory VaR* solution set that are weakly dominated by at least one portfolio in the *Mean-CR* Pareto-optimal front. We postulate that only those weakly dominated *Mean-Regulatory VaR* portfolios are improved by *Mean-CR* optimization. For the weakly dominated portfolios, we determine the closest dominant portfolio in the *Mean-CR* Pareto-optimal front in terms of the Euclidean distance. In doing so, we consider that *CR* and returns are not of the same order of magnitude. Thus, before calculating the Euclidian distance between any two portfolios, we divide each of the portfolio's objective values by its corresponding maximum value. We then calculate the *CR* and return differences between each weakly dominated *Mean-Regulatory VaR* portfolio and its closest dominant portfolio on the *Mean-CR* Pareto-optimal front. Correlation differences, *CR* improvements, return improvements and number of improved portfolios in different stress scenarios are presented in Table 5.

**Table 5.** Correlation differences, *CR* improvements, return improvements and number of improved portfolios

	High Volatility				Low Volatility			
	Correlation differences (%)	<i>CR</i> Improvement (%)	Return improvements (%)	<i>N</i>	Correlation differences (%)	<i>CR</i> improvements (%)	Return improvements (%)	<i>N</i>
<b>Historical Stress</b>	18.69	1.68	0.28	50	95.20	3.98	0.76	86
<b>HSS1</b>	0.00	0.32	0.13	18	0.00	0.14	0.12	21
<b>HSS2</b>	0.00	0.11	0.24	47	0.00	0.14	0.12	19
<b>HSS3</b>	52.26	6.19	1.06	76	105.04	6.81	1.72	65

The largest differences between the correlation matrices of the original and stressed returns are observed for historical and *HSS3* scenarios, particularly in the low-volatility sample. It is

precisely in these two scenarios that the *Mean-CR* optimization significantly outperforms the *Mean-Regulatory VaR* benchmark.

Consistent with the results in Section 7.2, the greatest improvement in *CR* and returns are achieved for the historical and *HSS3* stress scenarios. Improvements are more pronounced in the low-volatility sample than in the high-volatility sample. Note that the historical stress scenario is the same for both high- and low-volatility samples. However, in the low-volatility environment, the original correlations are smaller than in the high-volatility environment. This leads to greater differences between the original and stressed correlations and, consequently, to greater improvements in *Mean-CR* trade-offs. Under *HSS3*, correlations are doubled but capped at 0.95 independent of the volatility regime. In the low-volatility environment, where the original correlations are relatively small, this typically leads to doubling of the original correlations. In contrast, in the high-volatility environment, the original correlations are higher and thus the cap is reached much more often. As a result, the value of correlation differences under the *HSS3* scenario in the high-volatility sample is roughly half the value reached in the low-volatility sample (52.26% versus 105.04%). However, it is also much higher than the value for the historical scenario in the high-volatility sample (18.69%) (see Table 5).

For robustness checks, we also compared the *Mean-CR* and *Mean-Regulatory VaR* solutions subject to the four stress scenarios using the  $\varepsilon$ -indicator (Zitzler et al., 2003) and generational distance (*GD*) (see Van Veldhuizen and Lamont, 1998; Van Veldhuizen and Lamont, 2002). The results presented in the Appendix are consistent with the results presented in Table 5.

### 6.3.2. Cardinality of portfolios

In Figures 13 and 14 we present cardinality (the number of assets) for portfolios entering *Mean-CR*, *Mean-Regulatory VaR* and *Mean-VaR* Pareto-optimal sets in the high- and low-volatility samples when the historical stress scenario is applied. Here, we consider only assets with weights greater than 2%.<sup>4</sup>

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<sup>4</sup> For robustness checks, we also tried a 1% threshold. The results are consistent with the reported results for the higher threshold of 2%.



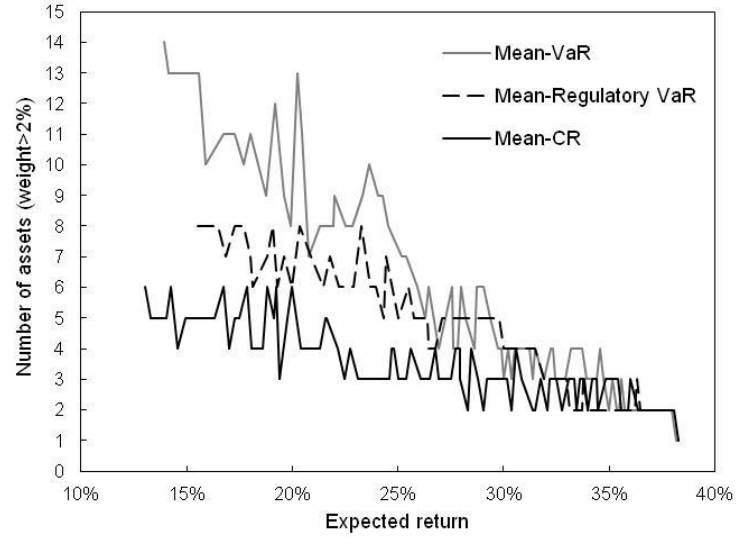


Figure 13. Portfolio cardinality for historical stress scenario: low-volatility sample

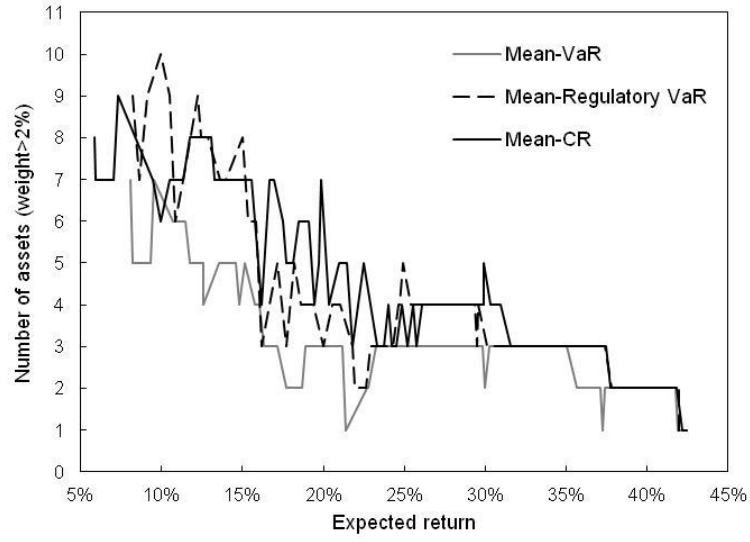


Figure 14. Portfolio cardinality for historical stress scenario: high-volatility sample

In Figures 15 and 16, we present the number of assets included in the *Mean-CR*, *Mean-Regulatory VaR* and *Mean-VaR* Pareto-optimal set, in the high- and low-volatility samples when the *HSS3* stress scenario is applied.

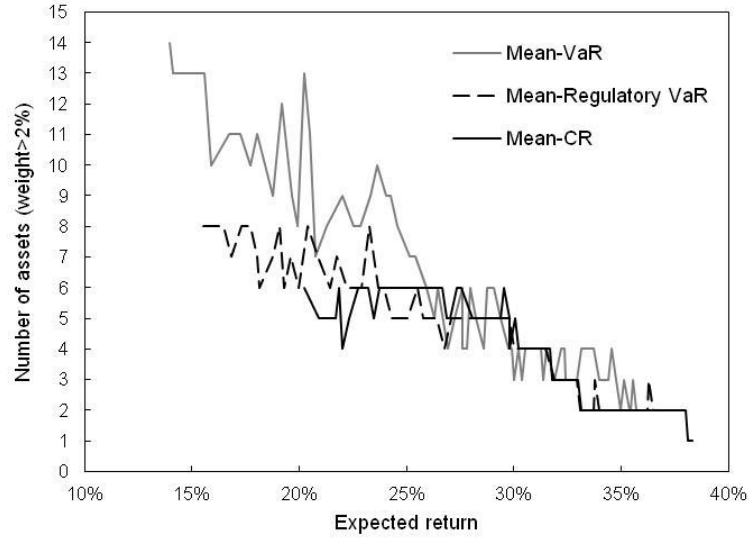


Figure 15. Portfolio cardinality for *HSS3* stress scenario: low-volatility sample

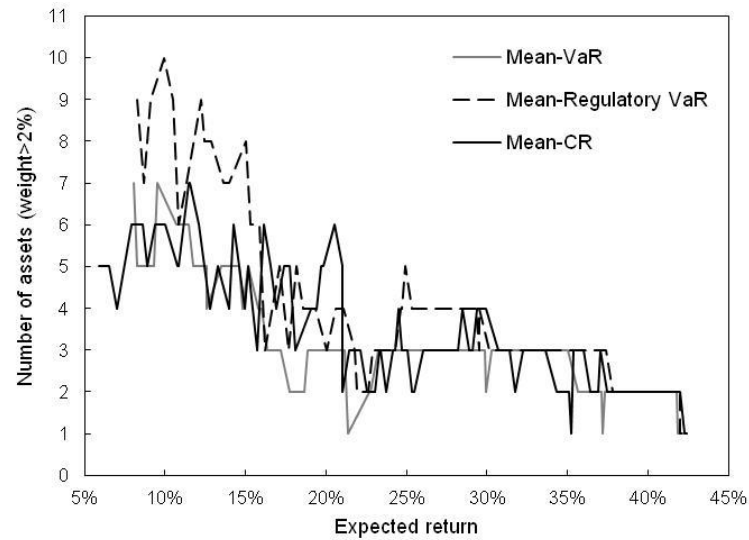


Figure 16. Portfolio cardinality for *HSS3* stress scenario: high-volatility sample

The results presented in Figures 13 to 16 suggest that portfolios with low returns (and low *CR*) contain a higher number of assets thus exhibiting higher diversification. The increase in returns (and therefore risk) results in a decrease in cardinality of Pareto-optimal portfolios (i.e. lower diversification).

Importantly, *Mean-CR* optimization is associated with a significant decrease in cardinality in the low-volatility sample both for the historical and *HSS3* stress scenarios, and for the *HSS3* stress scenario in the high-volatility sample. As stated earlier, these are the cases in which *Mean-CR* optimization outperforms *Mean-Regulatory VaR* benchmark the most. The greatest improvements with respect to that benchmark are therefore associated with a decrease in the cardinality of *Mean-CR* Pareto-optimal portfolios for comparable returns. Note that for the *HSS1* and *HSS2* stress scenarios, we detect neither a significant improvement in the *Mean-CR*

trade-offs nor a reduction in cardinality with respect to the benchmark *Mean-Regulatory VaR* optimization. These are stress scenarios for which correlations are not impacted.

### 6.3.3. An alternative opportunity set

We performed robustness checks using the opportunity set of Morgan Stanley Composite indices (*MSCI*) for 31 countries.<sup>5</sup> To determine the maximum and minimum volatility dates, we apply the same approach as for the *S&P100* sample. Specifically, for each day in the rolling estimation period of 421 days (from January 4, 2012 to September 6, 2013), we estimated the 1-day-ahead daily returns volatility of the *MSCI World* US dollar-denominated price index. This index serves as a market portfolio proxy in this case. A maximum volatility of 0.013682 (20.46% in annual terms) is determined on June 29, 2012, whereas a minimum volatility of 0.006662 (9.87% in annual terms) is determined on August 14, 2013. The unreported results are statistically and economically consistent with the results obtained for the *S&P 100* stocks.<sup>6</sup>

## 7. Conclusions

We propose a novel method for *Mean-CR* portfolio optimization that accurately incorporates Basel 2.5 regulation for market risk capital requirements in the optimization procedure. In our approach, *CR* is calculated within the actual portfolio framework whereas the corresponding *VaR* estimation is based on the univariate *GARCH VaR* analytical model. We solve the optimization problem by employing the *NSGA-II* algorithm within *WoBinGO*, a parallel framework for genetic algorithm-based optimization. The method is applied to the opportunity set consisting of the 40 largest stocks in the *S&P 100* index. Our findings are confirmed in unreported robustness tests on the investment universe consisting of the 31 *MSCI* country indices.

We compare the results of *Mean-CR* optimization with two simpler optimization approaches, namely *Mean-VaR* and *Mean-Regulatory VaR*. Our results provide several important insights for financial institutions and regulators. First, *Mean-VaR* optimized portfolios are a relatively

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<sup>5</sup> The 31 market indices are US dollar-denominated price indices that include large and mid-cap securities from the following countries: Australia, Brazil, Canada, Chile, Colombia, the Czech Republic, Denmark, Hungary, India, Indonesia, Israel, Japan, Korea, Malaysia, Mexico, Morocco, New Zealand, Norway, Peru, the Philippines, Poland, Russia, Singapore, South Africa, Sweden, Switzerland, Taiwan, Thailand, Turkey, the United Kingdom and the United States.

<sup>6</sup> The results are available from the authors upon request.

poor proxy for the optimal trade-off between portfolio returns and corresponding capital charges under both Basel II and Basel 2.5 regulatory frameworks. Second, unlike *Mean-VaR* optimal portfolios, *Mean-Regulatory VaR* optimization generates optimal portfolios that never lead to a non-zero penalty factor  $k$ . Furthermore, our optimization approach is successful in finding portfolios that keep the number of *VaR* violations within the no-penalty zone (in addition to portfolios which are *Mean-Regulatory VaR*-efficient) while reducing *Regulatory Stressed VaR*.

For the stress tests that do not change correlations, adding the *Regulatory Stressed VaR* term to the optimization process does not substantially impact *Mean-CR* trade-offs. If financial institutions were to apply such tests, they could simplify the optimization procedure by effectively ignoring the stress term in the optimization problem and consequently, keeping their portfolios more diversified. In contrast, *Mean-CR* optimal portfolios tend to outperform *Mean-Regulatory VaR* counterparts when the stress test substantially impacts correlations. These improvements, measured by average *CR* and return differences, are related to a decrease in the cardinality of portfolios with comparable returns. This is particularly evident for portfolios with lower expected returns. These results are consistent with anecdotal evidence showing the reduced benefits of diversification in the aftermath of the recent financial crisis. Namely, if all banks aim to minimize their capital charge, they are likely to hold few assets in their portfolios, adversely impacting the liquidity of all other assets. Therefore, our study informs the ongoing debate about the implementation of a new regulatory framework (Basel III) scheduled for 2019.

## Appendix

### A.1 $\varepsilon$ -indicator

The  $\varepsilon$ -indicator determines the minimum value by which a reference set must be multiplied so that every solution in the reference set becomes weakly dominated by at least one solution in the approximation set (see Zitzler et al., 2003). If the approximation set exactly matches the reference set, then the  $\varepsilon$ -indicator takes a value of one. In Table A1, the  $\varepsilon$ -indicator refers to the *Mean-Regulatory VaR* Pareto-optimal set presented in the *Mean-CR* space, whereas the *Mean-CR* Pareto-optimal front is used as the reference set. *Mean-Regulatory VaR* optimal solutions that became dominated in *Mean-CR* solution space have been discarded.

**Table A1.**  $\varepsilon$ -indicators and average correlation differences

	High Volatility		Low Volatility	
	$\varepsilon$	Correlation differences (%)	$\varepsilon$	Correlation differences (%)
Historical Stress	1.054	18.69	1.148	95.20
<i>HSS1</i>	1.021	0.00	1.008	0.00
<i>HSS2</i>	1.022	0.00	1.009	0.00
<i>HSS3</i>	1.158	52.26	1.300	105.04

### A.2 Generational distance (*GD*)

The Generational Distance (*GD*) proposed by Van Veldhuizen and Lamont (1998) and Van Veldhuizen and Lamont (2002) determines the average Euclidian distance between solutions that belong to the known Pareto-optimal front and solutions belonging to the true Pareto-optimal front:

$$GD = \frac{\left( \sum_{i=1}^n d_i^2 \right)^{1/2}}{n} \quad (12)$$

where  $d_i$  denotes Euclidian distance between  $i$ -th solution of the known Pareto-optimal front and the closest solution of the true Pareto-optimal solution and,  $n$  denotes the number of solutions in the known Pareto-optimal front. Lower *GD* values suggest greater proximity between the known and true Pareto-optimal front.

Here, for the known Pareto-optimal front, we adopt the *Mean-Regulatory VaR* Pareto-optimal set presented in the *Mean-CR* space, whereas for the true Pareto-optimal front, we use the *Mean-CR* Pareto-optimal front. Again, *Mean-Regulatory VaR* optimal solutions that became

dominated in *Mean-CR* solution space have been discarded. We normalize the portfolios' *CR* and return values by dividing them by the corresponding maximum values. In Table A2, we present *GD* values and average correlation differences in alternative scenarios.

**Table A2.** *GD* values and average correlation differences

	High Volatility		Low Volatility	
	<i>GD</i>	Correlation differences (%)	<i>GD</i>	Correlation differences (%)
<i>Historical Stress</i>	9.38E-4	18.69	3.40E-3	95.20
<i>HSS1</i>	4.82E-4	0.00	3.53E-4	0.00
<i>HSS2</i>	6.54E-4	0.00	3.52E-4	0.00
<i>HSS3</i>	5.35E-3	52.26	6.97E-3	105.04

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